

# **New Design Approach for Axially Compressed Composite Cylindrical Shells combining the Single Perturbation Load Approach and Probabilistic Analyses**

March 25, 2015

DESICOS Conference on Buckling and Postbuckling Behaviour of  
Composite Structures, Braunschweig

Alexander Meurer, Mona Dannert and Raimund Rolfes

# **New Design Approach for Axially Compressed Composite Cylindrical Shells combining the Single Perturbation Load Approach and Probabilistic Analyses**

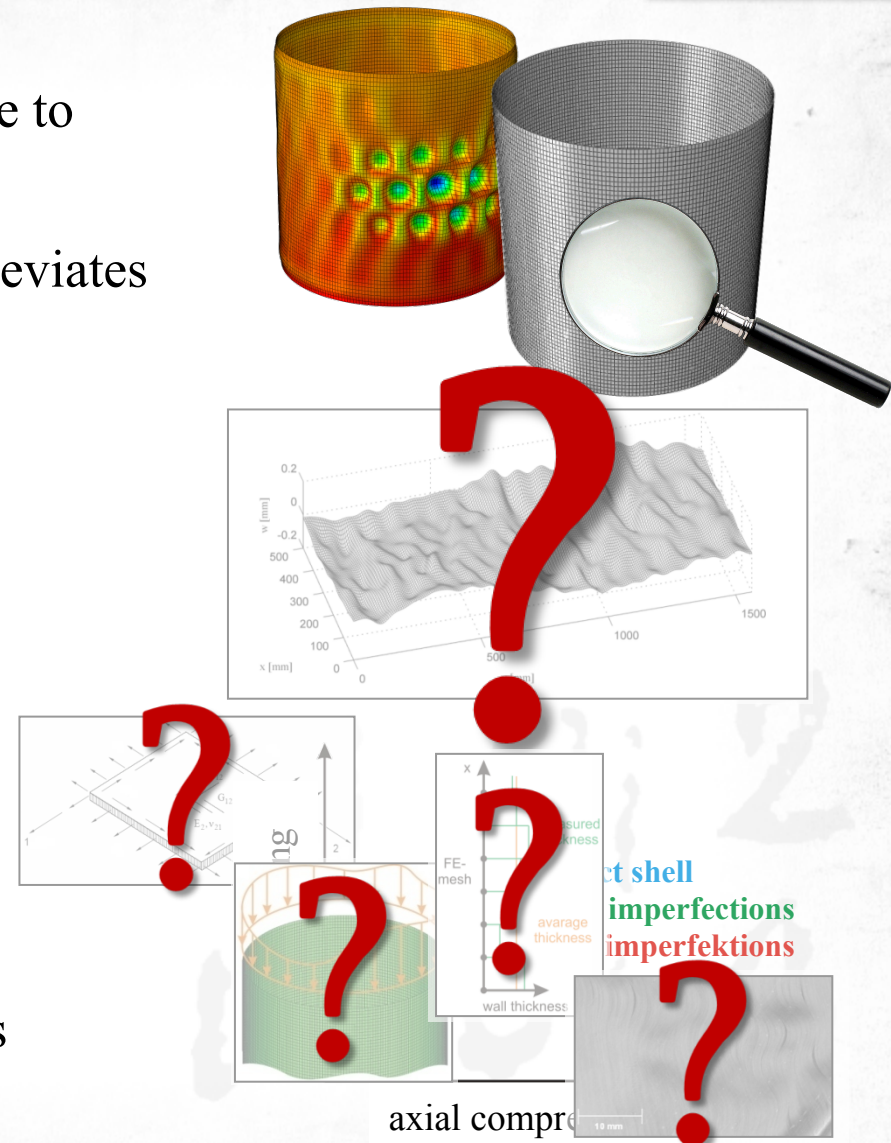
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# Introduction

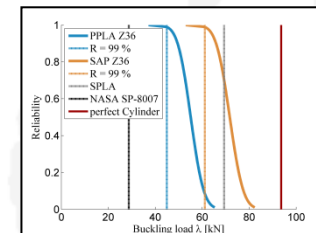
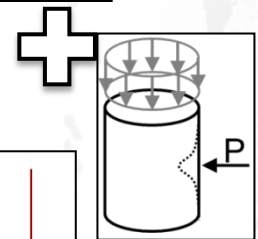
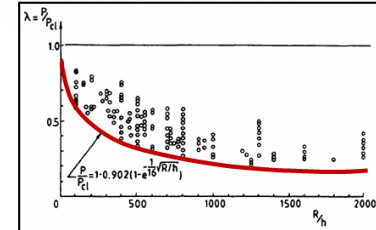


- ◆ Axially compressed cylinders are prone to buckling
- ◆ A real manufactured cylinder always deviates from the nominal structure
- ◆ These imperfections heavily affect the buckling load
  - ◆ traditional imperfections
  - ◆ non-traditional imperfections
    - ◆ material imperfections
    - ◆ uneven loading
    - ◆ thickness imperfections
    - ◆ ...
- ◆ Main problem in design: Imperfections are not known prior to manufacturing!



# Overview

- ◆ Introduction
- ◆ Design philosophies
- ◆ The “Probabilistic Perturbation Load Approach” (PPLA)
  - ◆ Semi-Analytical Probabilistic Procedure (SAP): Overview
  - ◆ PPLA: Main idea
  - ◆ Application to DESICOS use cases
  - ◆ Comparison to other design procedures
- ◆ Conclusion and next steps

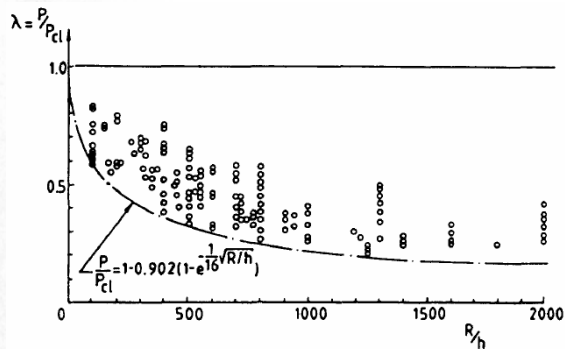




# Design philosophies

## Knock-Down-Factors

- ◆ NASA SP-8007

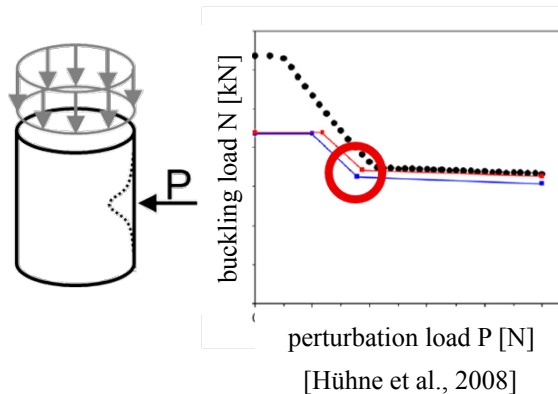


- ◆ based on buckling experiments
- ◆ in most cases overly conservative
- ◆ only partly applicable to composites



## Deterministic Design

- ◆ Single Perturbation Load Approach (SPLA)

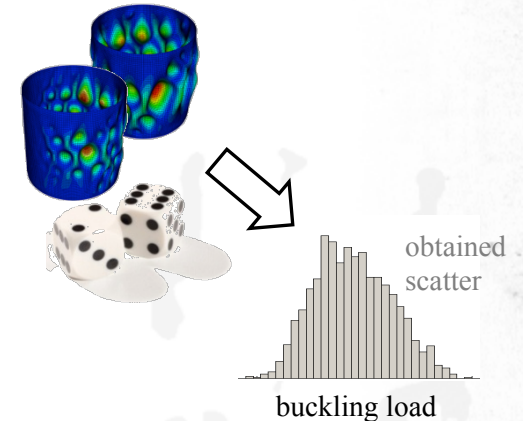


- ◆ no imperfection information necessary
- ◆ not always robust (with respect to the experimental buckling loads)



## Probabilistic Design

- ◆ Monte Carlo
- ◆ Semi-analytical procedures



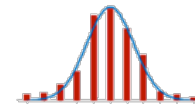
- ◆ known level of reliability
- ◆ imperfection information necessary
- ◆ Monte Carlo computationally costly



# Design philosophies

## ◆ Qualities of desired design procedure

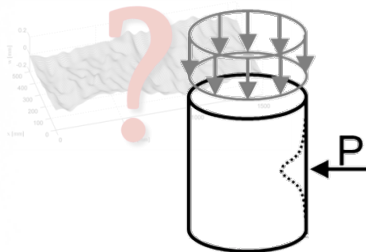
- ◆ no geometric imperfection information necessary
- ◆ incorporate scatter of non-traditional imperfections
- ◆ always robust



## ◆ Combination of

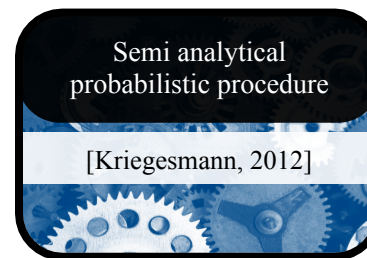
**Deterministic Design**

geometric imperfections



**Probabilistic Design**

scattering non-traditional imperfections



$f_t$

$f_{E_{11}}$

$f_{\Theta}$



$\lambda_d$

# Semi-Analytical Probabilistic Procedure (SAP)

[Kriegesmann, 2012]



- ◆ Taylor approximation of the objective function  
(= buckling load depending on scattering imperfections)

$$g(\mathbf{x}) = g(\boldsymbol{\mu}) + \sum_{i=1}^n \frac{\partial g(\boldsymbol{\mu})}{\partial x_i} (x_i - \mu_i) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 g(\boldsymbol{\mu})}{\partial x_i \partial x_j} (x_i - \mu_i)(x_j - \mu_j) + \dots$$

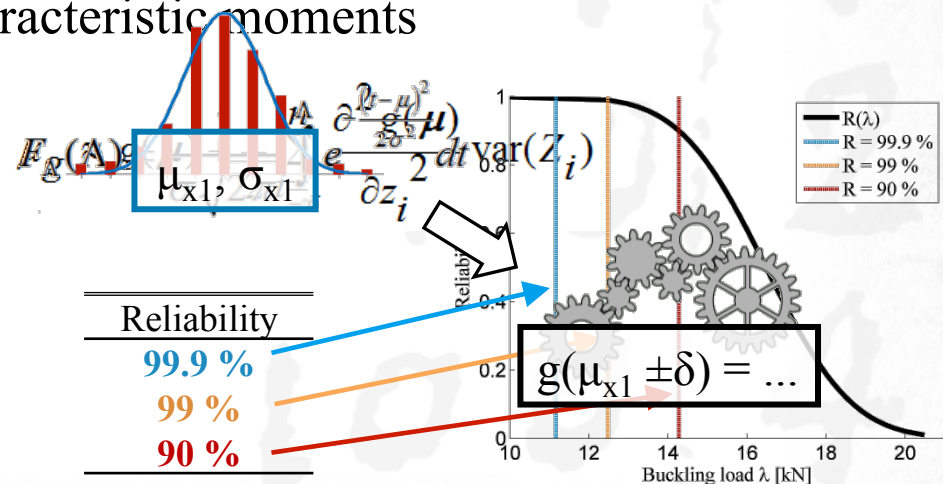
↳ Mahalanobis transformation to reduce and decorrelate the input parameters

- ◆ Evaluation of the objective function around the mean values of the input parameters  
 $\mathbf{x} = \boldsymbol{\Sigma}^{\frac{1}{2}} \mathbf{z} + \boldsymbol{\mu}$  and  $\mathbf{z} = \boldsymbol{\Sigma}^{-\frac{1}{2}} (\mathbf{x} - \boldsymbol{\mu})$

- ◆ Numerical determination of the characteristic moments

- ◆ Choice of a type of distribution  
(i.e. normal distribution)

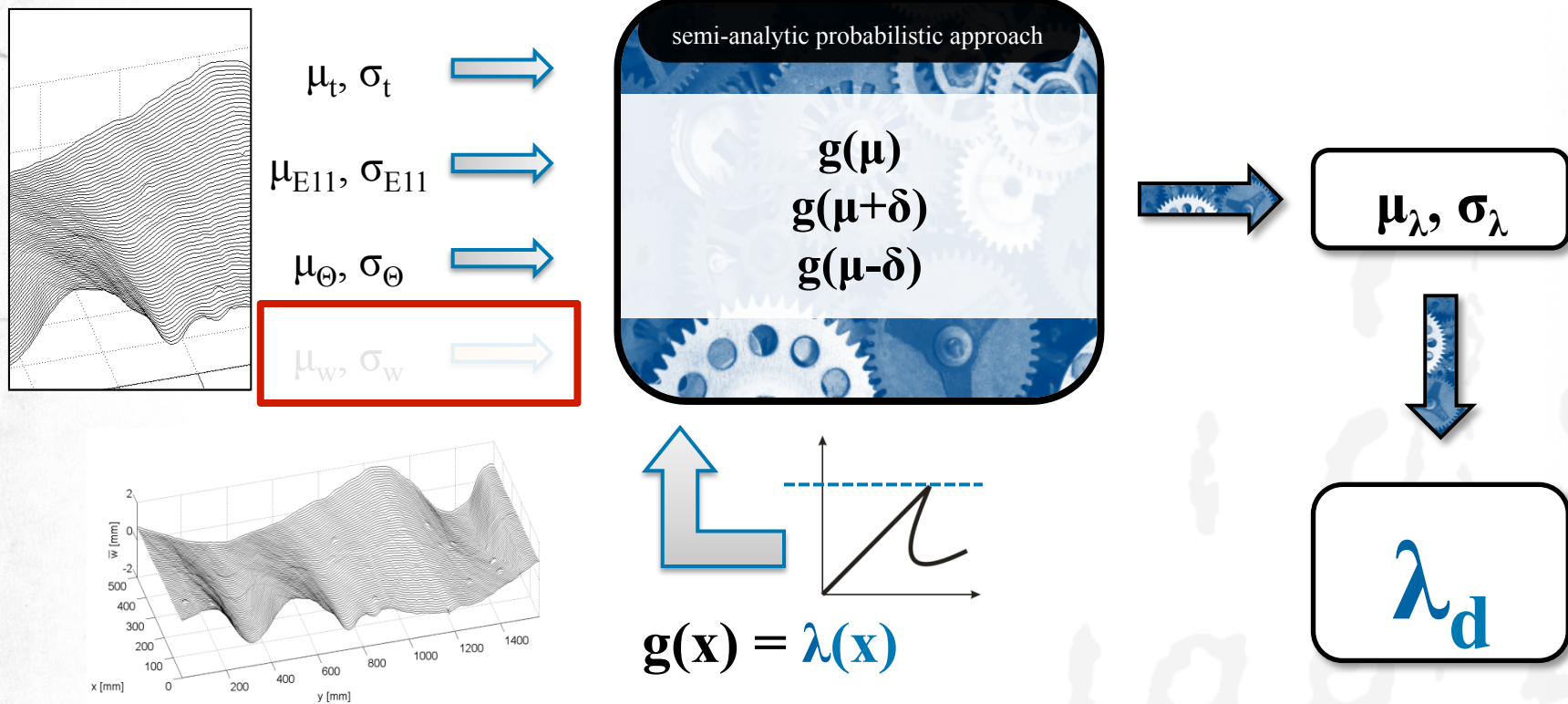
- ◆ Choice of a level of reliability to obtain a robust design load



# Probabilistic Perturbation Load Approach (PPLA)

Main idea:

◆ Combination of probabilistic and deterministic design approaches

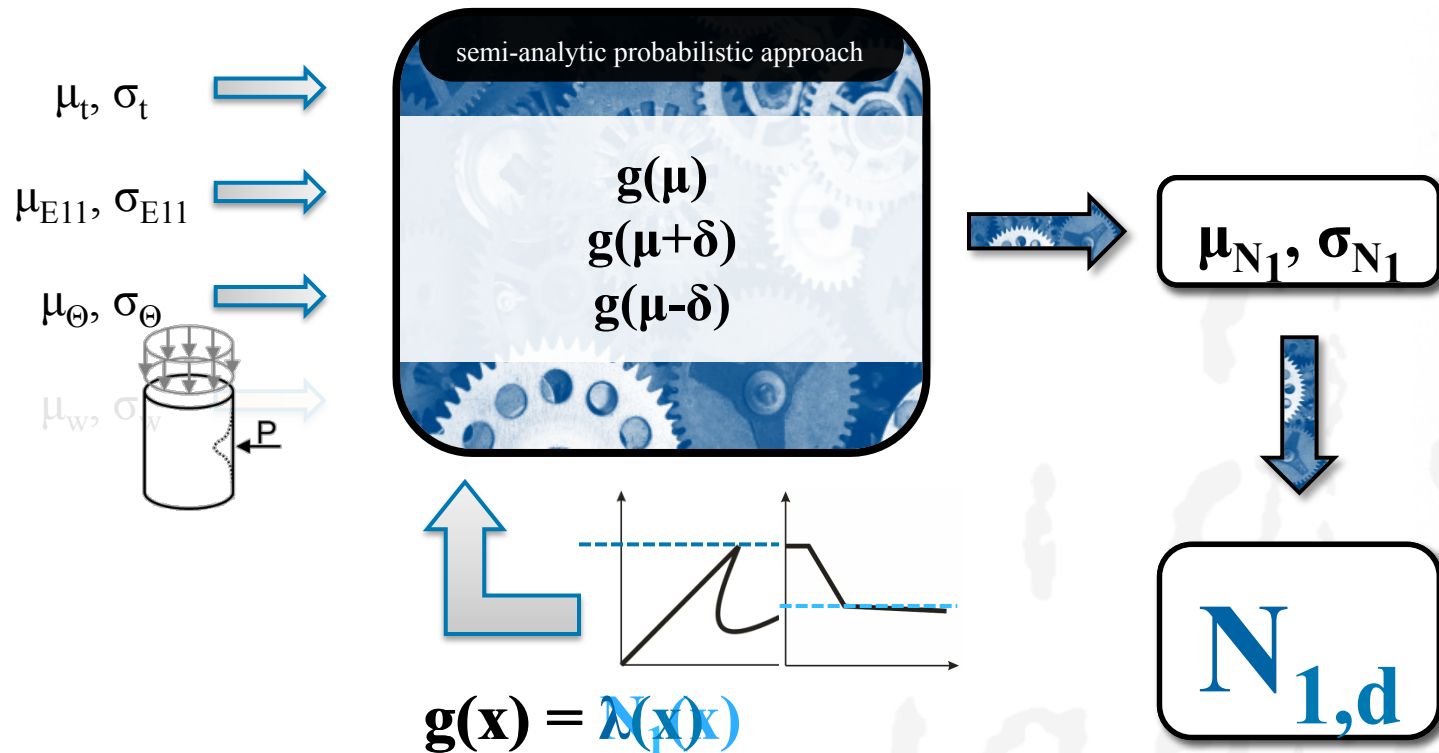




# Probabilistic Perturbation Load Approach (PPLA)

Main idea:

◆ Combination of probabilistic and deterministic design approaches



# PPLA: Application – Use Cases

## PPLA applied to shells treated within DESICOS

### ◆ Shell Z15 (DESICOS benchmark shell)

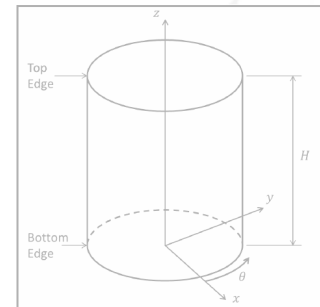
- ◆ nominal thickness  $t = 0.5$  mm
- ◆ free length  $L = 500$  mm, radius  $R = 250$  mm
- ◆ CFRP IM7/8552,  $[\pm 24 / \pm 41]$



[Degenhardt et al., 2007]

### ◆ Shell Z36 (manufactured and tested within DESICOS)

- ◆ nominal thickness  $t = 0.75$  mm
- ◆ free length  $L = 800$  mm, radius  $R = 400$  mm
- ◆ CFRP IM7/8552,  $[\pm 34 / 0 / 0 / \pm 53]$



# PPLA: Application to Z15 – Input Parameters



## ◆ Scattering wall thickness

◆ Data basis: measurements of Cylinders Z15-Z26

$\mu_t, \sigma_t$

◆ Smeared wall thickness derivation for the entire shell

$\mu_{E11}, \sigma_{E11}$

## ◆ Scattering material properties

◆ Central moments obtained from [Degenhardt et al., 2009]

$\mu_{E22}, \sigma_{E22}$

$\mu_{G12}, \sigma_{G12}$

## ◆ Loading imperfections

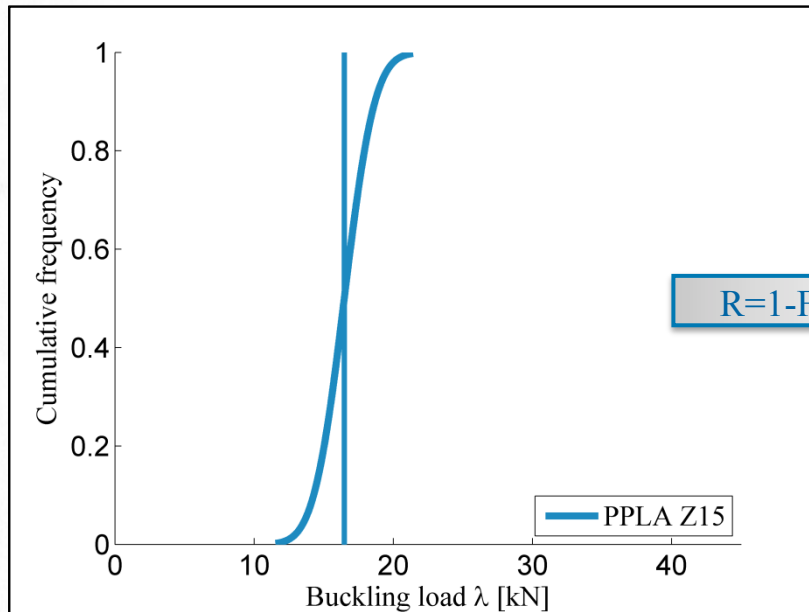
◆ Central moments obtained from [Kriegesmann, 2012]

$\mu_\Theta, \sigma_\Theta$

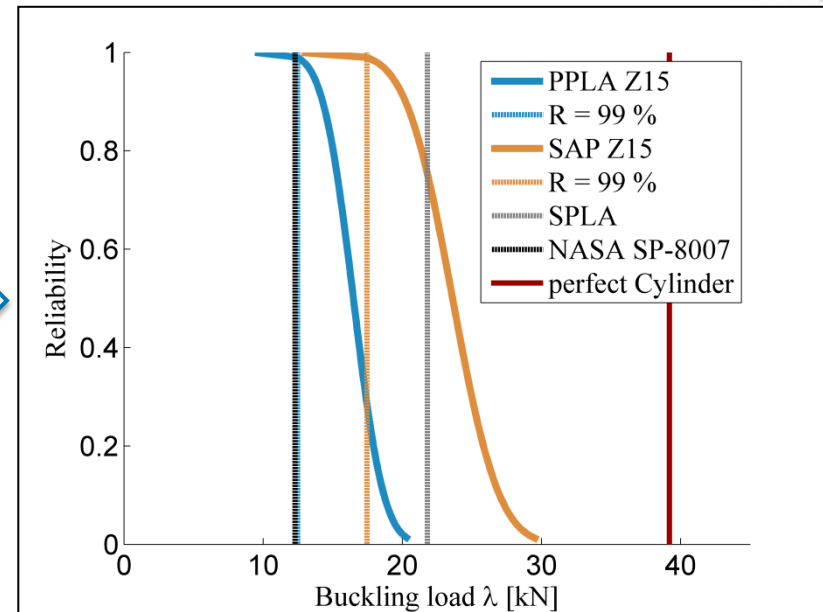
◆ Positioning of loading imperfections uniformly distributed on interval  $[0^\circ 180^\circ]$

$\mu_\omega, \sigma_\omega$

# PPLA: Application to Z15 – Results



$$R=1-F$$



Approach	Reliability	Design load	Knock-Down
<b>PPLA Z15</b>	<b>99 %</b>	<b>12.5 kN</b>	<b>0.32</b>
<b>SAP Z15</b>	<b>99 %</b>	<b>17.4 kN</b>	<b>0.44</b>
NASA SP-8007	-	12.3 kN	0.32
Min. test result		<b>21.3 kN</b>	<b>0.54</b>



# PPLA: Application – Use Cases

## PPLA applied to shells treated within DESICOS

### ◆ Shell Z15 (DESICOS benchmark shell)

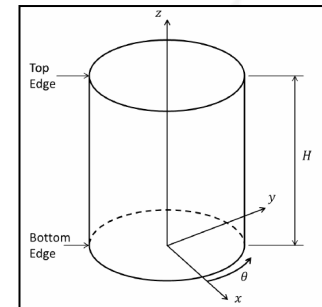
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[Degenhardt et al., 2007]

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# PPLA: Application to Z36 – Input Parameters



## ◆ Scattering wall thickness

◆ Data basis: measurements of Cylinders Z15-Z26

◆ Smeared mean thickness value for every shell obtained from [Degenhardt et al., 2009] leads to

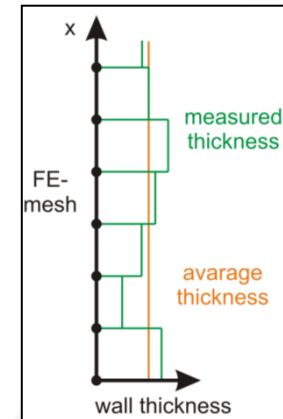
$$\Delta t_{\text{Degenhardt},i} = t_{\text{measured},i} - t_{\text{nom},Z15}$$

◆ Predicted mean thicknesses for new laminate setup:

$$t_i = t_{\text{nom},Z36} + \Delta t_{\text{Degenhardt},i}$$

◆ Determination of central moments of wall thickness

$$\mu_t, \sigma_t$$



# PPLA: Application to Z36 – Input Parameters



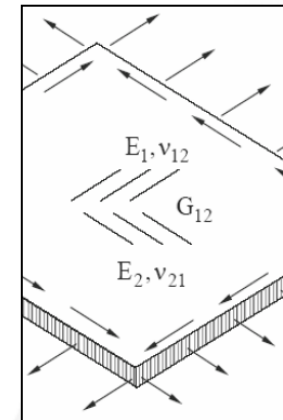
## ◆ Scattering material properties

- ◆ Same CFRP material as Z15, thus the ESA study can serve as data basis
- ◆ Central moments obtained from [Degenhardt et al., 2009]

$$\mu_{E11}, \sigma_{E11}$$

$$\mu_{E22}, \sigma_{E22}$$

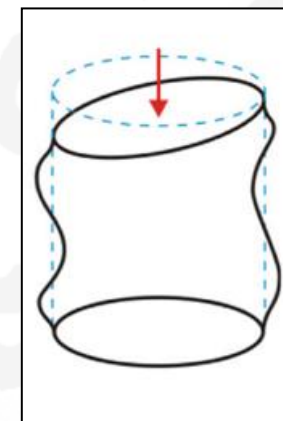
$$\mu_{G12}, \sigma_{G12}$$



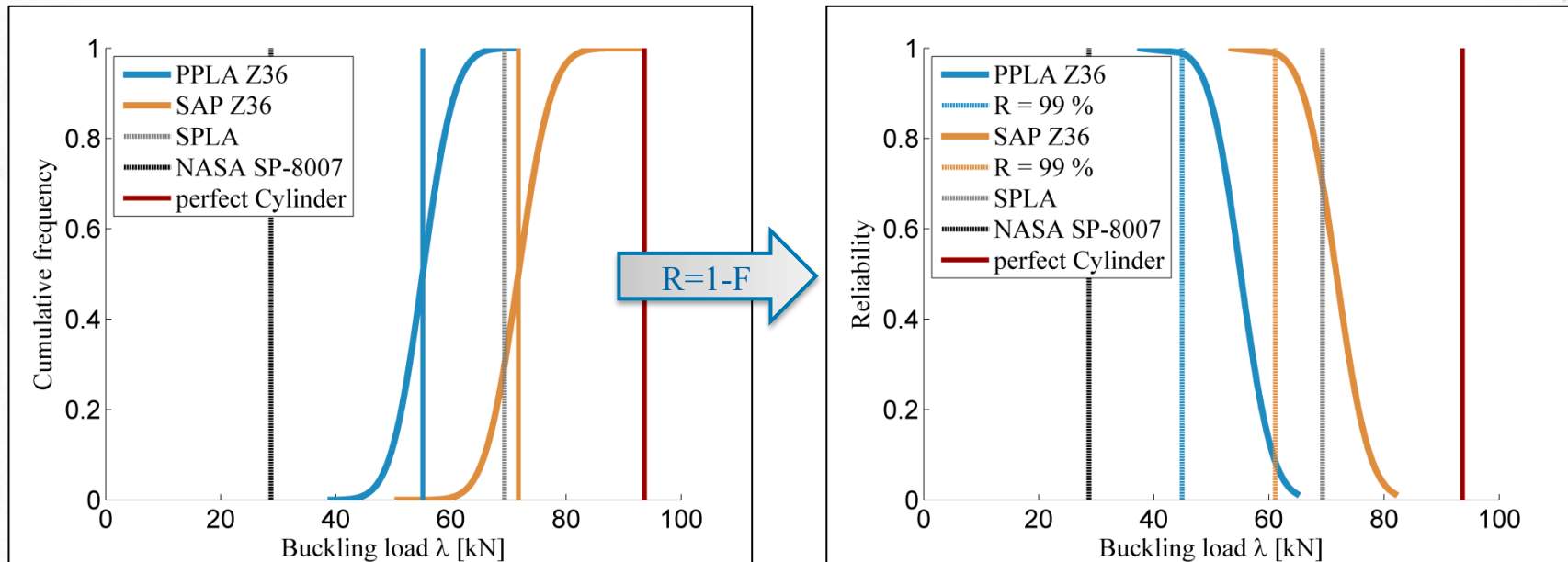
## ◆ Loading imperfections

- ◆ Same testing machine as in [Degenhardt et al., 2009]
- ◆ Central moments obtained from [Kriegesmann, 2012]

$$\mu_{\Theta}, \sigma_{\Theta}$$



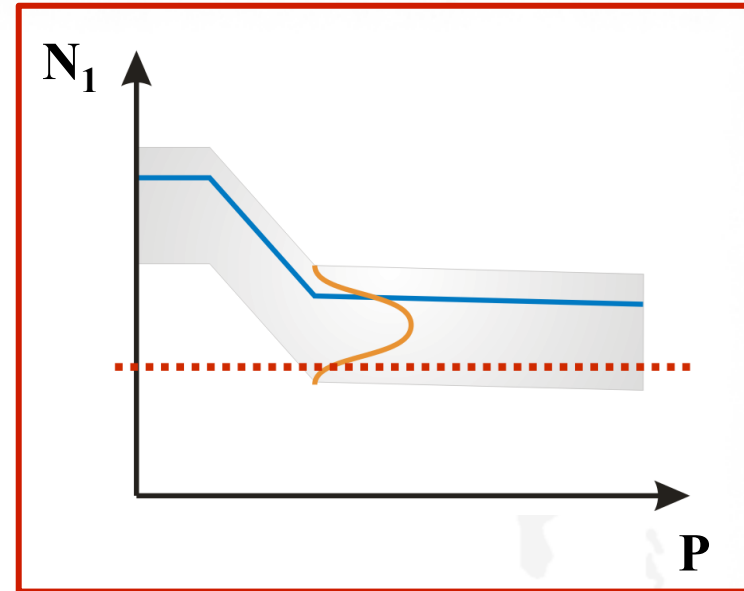
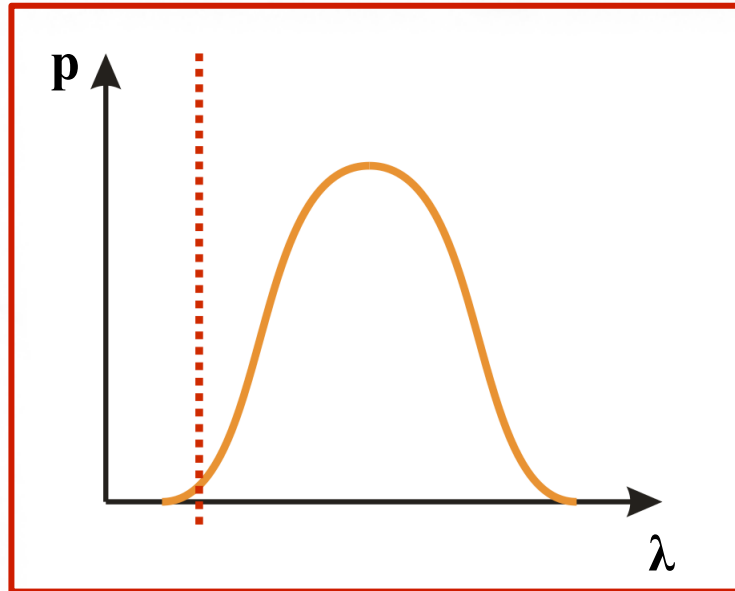
# PPLA: Application to Z36 – Results



Approach	Reliability	Design load	Knock-Down
<b>PPLA Z36</b>	<b>99 %</b>	<b>44.9 kN</b>	<b>0.48</b>
<b>SAP Z36</b>	<b>99 %</b>	<b>61.1 kN</b>	<b>0.65</b>
NASA SP-8007	-	28.7 kN	0.31
Test result		<b>64.0 kN</b>	<b>0.68</b>



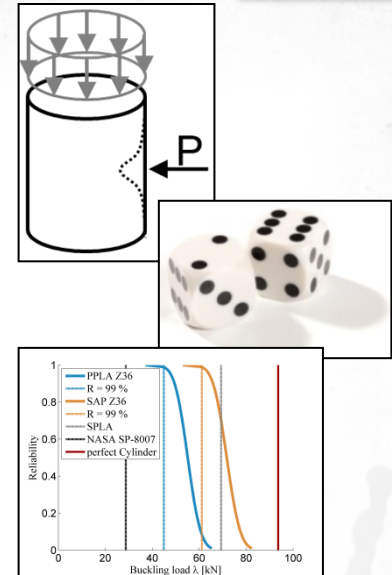
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# Conclusion and next steps

- ◆ A combination of the SPLA and a semi-analytical probabilistic procedure has been established
  - ◆ geometric imperfections are covered by SPLA
  - ◆ non-traditional imperfections are covered stochastically
  - ◆ robust design loads were obtained



## Future work:

- ◆ For which laminate setups is the SPLA applicable?
- ◆ Investigations on further non-traditional imperfections
- ◆ Comparison of the simulations with experimental results within DESICOS



[Hühne et al., 2008]

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# Backup



# PPLA – Evaluation of the objective function



## ◇ Taylor approximation of the objective function

$$g(\mathbf{x}) = g(\boldsymbol{\mu}) + \sum_{i=1}^n \frac{\partial g(\boldsymbol{\mu})}{\partial x_i} (x_i - \mu_i) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{\partial^2 g(\boldsymbol{\mu})}{\partial x_i \partial x_j} (x_i - \mu_i)(x_j - \mu_j) + \dots$$

SAP:  $g(\mathbf{x}) = \lambda(\mathbf{x})$       buckling load ✓ [Kriegesmann, 2012]  
PPLA:  $g(\mathbf{x}) = N_1(\mathbf{x})$       design load by SPLA

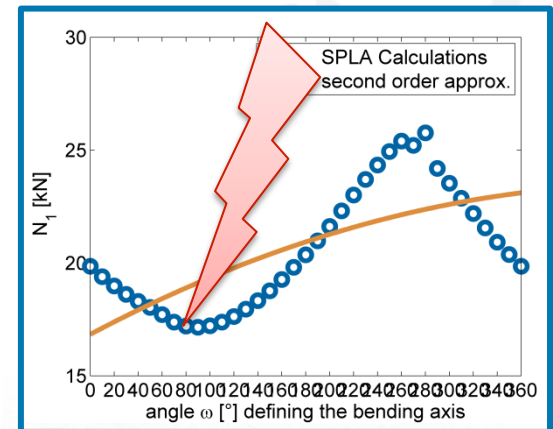
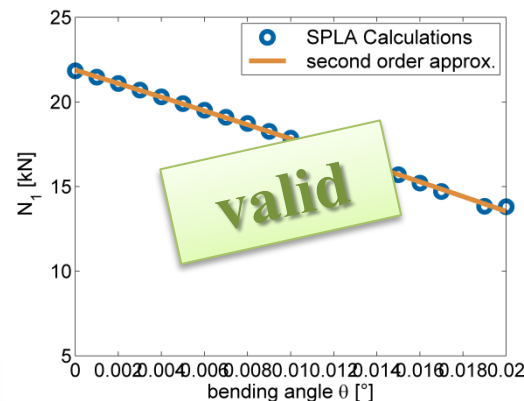
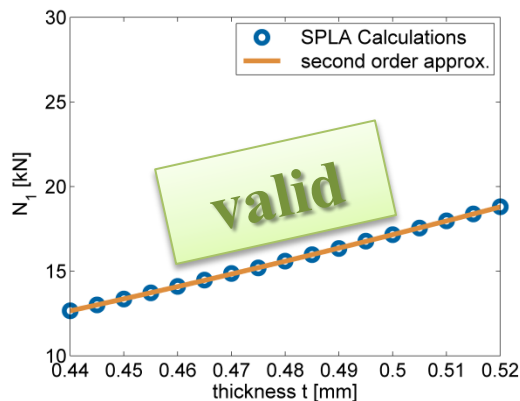
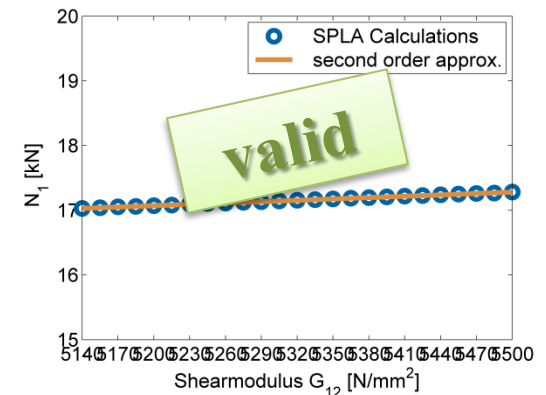
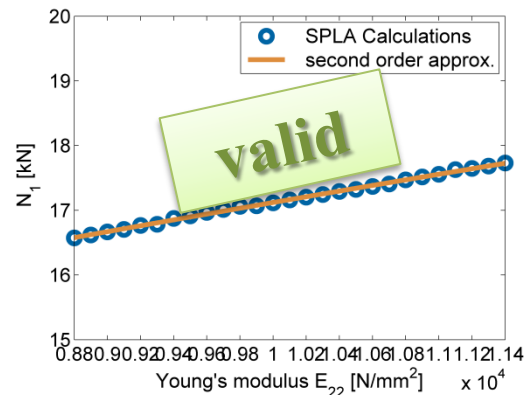
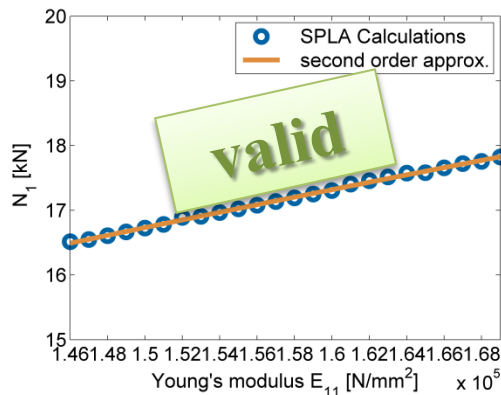


Is a second order Taylor approximation still valid for  $N_1(\mathbf{x})$  ?

# PPLA – Evaluation of the objective function

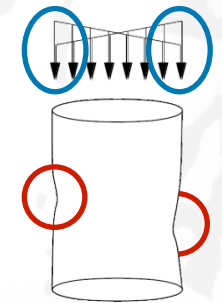
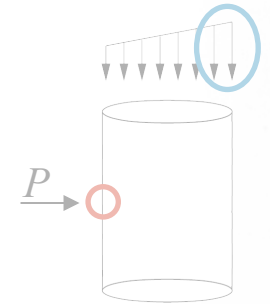
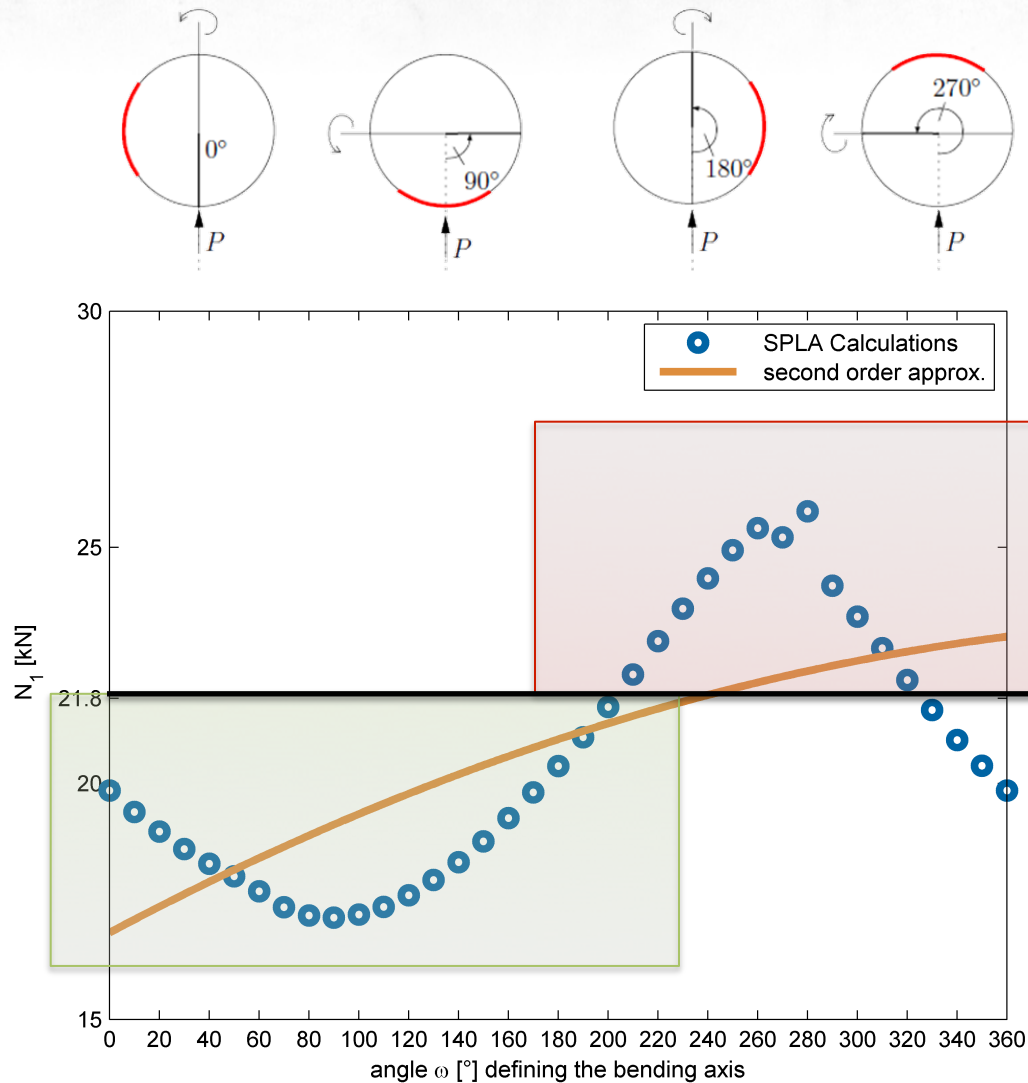
◇  $N_1(\mathbf{x})$  for  $x_i = [\mu_{x,i} - 3\sigma_{x,i} ; \mu_{x,i} + 3\sigma_{x,i}]$

Is a second order Taylor approximation still valid for  $N_1(\mathbf{x})$  ?



# PPLA – Evaluation of the objective function

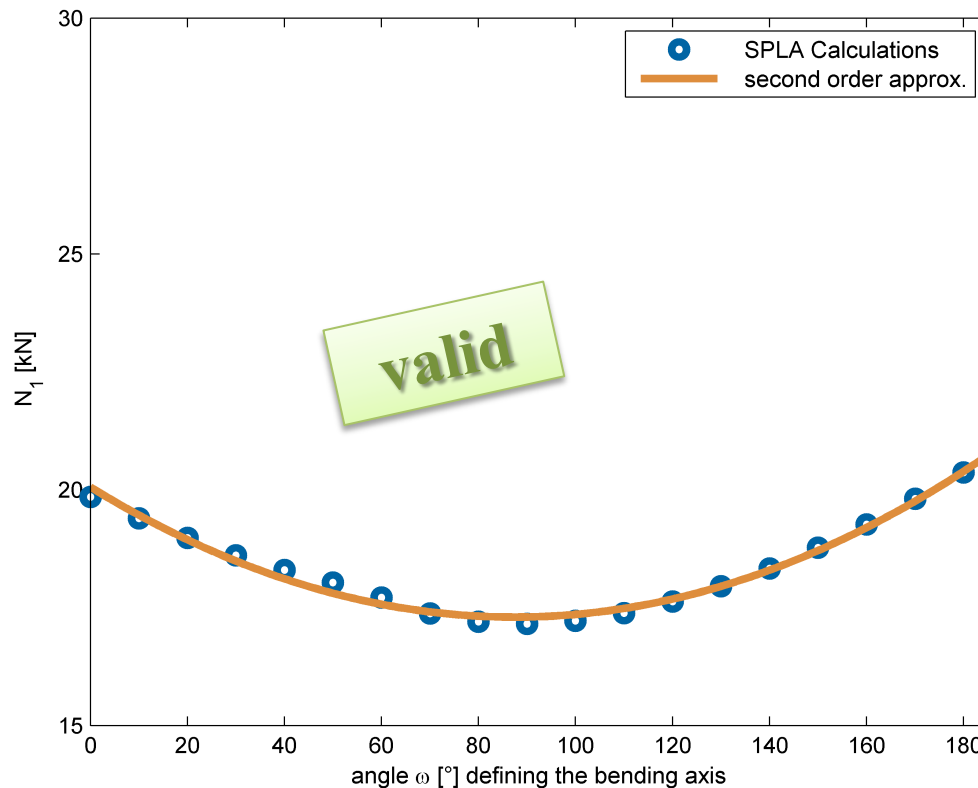
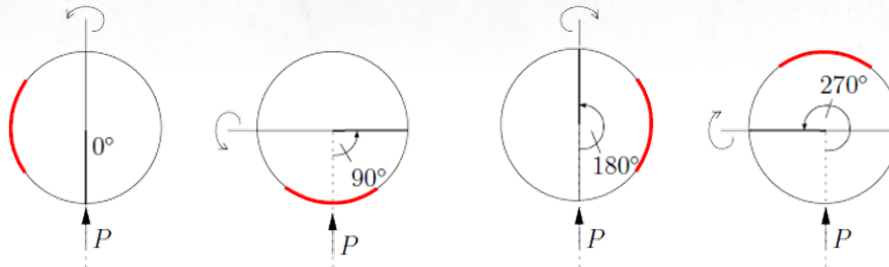
◆  $N_1(\omega)$



# PPLA – Evaluation of the objective function



◆  $N_1(\omega)$



Is a second order Taylor approximation still valid for  $N_1(\mathbf{x})$  ?